

Financial [114 marks]

1. [Maximum mark: 6]

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r .

(a) Find the value of r , giving your answer to four significant figures. [3]

(b) Laurie makes no further deposits to or withdrawals from the account.

Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

2. [Maximum mark: 12]

Helen and Jane both commence new jobs each starting on an annual salary of $\$70,000$. At the start of each new year, Helen receives an annual salary increase of $\$2400$.

Let $\$H_n$ represent Helen's annual salary at the start of her n th year of employment.

(a) Show that $H_n = 2400n + 67600$. [2]

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

Jane's annual salary, $\$J_n$, at the start of her n th year of employment is given by $J_n = 70000(1.03)^{n-1}$.

- (b) Given that J_n follows a geometric sequence, state the value of the common ratio, r . [1]

At the start of year N , Jane's annual salary exceeds Helen's annual salary for the first time.

- (c.i) Find the value of N . [3]

- (c.ii) For the value of N found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar. [2]

- (d) Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar. [4]

3. [Maximum mark: 12]

Andy and Jess each have \$5000.

Andy invests the money in a new savings plan that will pay interest at the end of each month.

Andy will receive a fixed amount of interest each month. The amount received is 0.315% of the initial investment.

- (a.i) Determine the amount of interest Andy will receive at the end of each month. Give this answer correct to two decimal places. [1]

- (a.ii) Hence, determine the amount of interest Andy will receive each year. [1]

- (a.iii) Write down an expression in the form $5000 + pn$, where $p \in \mathbb{Z}^+$, for the amount of money Andy will have in his savings plan at the end of n years. [1]

(a.iv) Hence, show that Andy will have \$5945 in his savings plan at the end of 5 years.

[1]

In this part, where appropriate, give all answers to the nearest dollar.

Jess invests her \$5000 in a new account that pays 3% interest compounded annually.

(b.i) Determine the amount of money that will be in Jess's account at the end of 5 years.

[2]

(b.ii) Hence, find the amount of interest Jess will receive in the 5 years.

[1]

(c.i) Write an expression in the form $5000 \times q^n$, where $q \in \mathbb{R}^+$, for the amount of money that Jess will have in her account at the end of n years.

[2]

(c.ii) Hence, determine the smallest number of complete years that it will take for Jess to have more money in her account than Andy has in his savings plan.

[3]

4. [Maximum mark: 7]

Bob invests 1 000 dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1\,000(1 + k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k .

[1]

(b) Expand and simplify $(1 + x)^4$.

[2]

- (c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

5. [Maximum mark: 7]

Darren buys a car for \$35 000. The value of the car decreases by 15% in the first year.

- (a) Find the value of the car at the end of the first year. [2]

After the first year, the value of the car decreases by 11% in each subsequent year.

- (b) Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar. [2]

When Darren has owned the car for n complete years, the value of the car is less than 10% of its original value.

- (c) Find the least value of n . [3]

6. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is \$4200.

Option B The first payment is \$1500. In each month which follows, the payment is 4% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(a.i) Option A; [2]

(a.ii) Option B. [3]

Sorin received an inheritance of \$160 000. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 5% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years. [1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c.i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month. [3]

(c.ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least \$257 000. [3]

At the end of the three years, Daniela invested \$30 000 for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to \$41 000 after six years. [3]

7. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is \$5500.

Option B The first payment is \$2000. In each month which follows, the payment is 6% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(a.i) Option A; [2]

(a.ii) Option B. [3]

Sorin received an inheritance of \$120 000. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 4% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years. [1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c.i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month. [3]

(c.ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least \$250 000. [3]

At the end of the three years, Daniela invested \$40 000 for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

- (d) Find the value of r if this investment grows to \$53 000 after six years. [3]

8. [Maximum mark: 6]

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in USD (\$).

- (a) Write down the annual rate of depreciation of the car. [1]
- (b) Find the value of the car on 1 January 2028. [2]

Alvie wants to buy this car. On 1 January 2023, he invested \$15 000 in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra \$ M .

- (c) Find the value of M . [3]

9. [Maximum mark: 6]

In this question, give all answers correct to two decimal places.

Sam invests \$1700 in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

- (a) Find the amount that Sam will have in his account after 10 years. [3]

David also invests \$1700 in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or

withdrawals from this account.

(b) Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. [2]

(c) Find the interest David will earn over the 10 years. [1]

10. [Maximum mark: 6]

Gemma and Kaia started working for different companies on January 1st 2011.

Gemma's starting annual salary was \$45 000, and her annual salary increases 2% on January 1st each year after 2011.

(a) Find Gemma's annual salary for the year 2021, to the nearest dollar. [3]

Kaia's annual salary is based on a yearly performance review. Her salary for the years 2011, 2013, 2014, 2018, and 2022 is shown in the following table.

year (x)	2011	2013	2014	2018	2022
annual salary (\$)	45 000	47 200	48 500	53 000	57 000

(b) Assuming Kaia's annual salary can be approximately modelled by the equation $S = ax + b$, show that Kaia had a higher salary than Gemma in the year 2021, according to the model. [3]

11. [Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving \$20 000. They each have \$9000 to invest.

Amelia invests her \$9000 in an account that offers an interest rate of 7% per annum compounded **annually**.

(a.i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars. [3]

(a.ii) Determine the number of years required for Amelia's investment to reach the target. [2]

(b) Bill invests his \$9000 in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

A third friend Chris also wants to reach the \$20 000 target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

(c.i) Show that Chris will never reach the target if his initial deposit is \$9000. [5]

(c.ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [3]

12. [Maximum mark: 6]

Give your answers in this question correct to the nearest whole number.

Imon invested 25 000 Singapore dollars (SGD) in a fixed deposit account with a nominal annual interest rate of 3.6%, compounded **monthly**.

(a) Calculate the value of Imon's investment after 5 years. [3]

- (b) At the end of the 5 years, Imon withdrew x SGD from the fixed deposit account and reinvested this into a super-savings account with a nominal annual interest rate of 5.7%, compounded **half-yearly**.

The value of the super-savings account increased to 20 000 SGD after 18 months.

Find the value of x .

[3]